SUSTAINABLE DEVELOPMENT SOLUTIONS NETWORK
A GLOBAL INITIATIVE FOR THE UNITED NATIONS

Preliminary Sustainable Development Goal (SDG) Index and Dashboard

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This paper and all supporting data are available online at
http://unsdsn.org/resources/publications/sdg-index/

The authors have prepared this draft document for public consultation to seek comments and suggestions for improvement by 31 March 2016. In particular, we are grateful for recommendations for additional data to be included in the SDG Index and its components. All comments and suggestions for additional data should be submitted through the online form. Following the public consultation, the SDSN will issue a revised and expanded version of this document for public use.

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Why develop an unofficial SDG Index and an SDG Dashboard?

The 17 Sustainable Development Goals (SDGs) adopted by all member states of the United Nations in September 2015 set ambitious objectives across the three dimensions of sustainable development – economic development, social inclusion, and environmental sustainability, underpinned by good governance. Sound metrics and data are critical for turning the SDGs into practical tools for problem-solving by (i) mobilizing governments, academia, civil society, and business; (ii) providing a report card to track progress and ensure accountability; and (iii) serving as a management tool for the transformations needed to achieve the SDGs by 2030 (SDSN 2015). Drawing on currently available data, this draft paper proposes a preliminary SDG Index and an SDG Dashboard that may help countries to mobilize stakeholders and identify priorities for early action. Both are unofficial measures that do not replace official statistics.

The UN Statistical Commission has initiated a process for developing a comprehensive, official indicator framework for the 17 SDGs and 169 targets. So far, the Inter-Agency and Expert Group on SDG Indicators (IAEG-SDGs) has identified an unwieldy list of some 230 indicators for adoption by the Statistical Commission in March 2016.[1] In comparison, the Millennium Development Goals (MDGs) used 60 globally harmonized indicators, although even this limited number of indicators has yet to be fully implemented. Most MDG indicators have suffered from large numbers of missing data points and some have been reported with lags of five years or more (Cassidy 2014). It will therefore take many years before an SDG indicator framework is underpinned by comprehensive data. In the meantime, interim measures are needed to promote the SDGs as practical tools for problem solving and to help countries identify priorities for early action.

The Bertelsmann Foundation, with support from the UN Sustainable Development Solutions Network (SDSN), issued a report (Kroll 2015), which was the first to propose an SDG Index for OECD countries as a short-hand way of tracking SDG achievement and determining priorities for implementation in each country. The report grapples with major challenges of consistent measurement across these countries. We applaud this important effort, and the SDSN was delighted to assist the Bertelsmann Foundation in its implementation. The report has rightly garnered significant attention by policymakers as well as the media.

Another significant effort has been undertaken by the Overseas Development Institute (ODI 2015), which presents a regional SDG Scorecard projecting trends across key dimensions of the SDGs to determine areas in which the fastest acceleration of progress will be required. The scorecard shows that business-as-usual trends will need to be reversed in many areas in order to achieve the SDGs. It covers all regions in the world, but relies on regional aggregates, so that its findings cannot be applied at the country level.

In this paper we propose, first, an initial country-level SDG Index for all developed and developing countries that measures SDG achievement across the 17 goals, using national cross-country data available today. Such an index will, in a highly preliminary way, rank countries across the SDGs to assess the current state of progress relative to peers (e.g. countries at a given income level or in a given geographic region). Second, we propose an SDG Dashboard that presents SDG data visually for each country and goal. Goals are highlighted in green, yellow, or red with red highlighting a country's most acute challenges. In this way, the Dashboard can help stakeholders identify the most urgent priorities in each country and region. It illustrates that even countries that rank highly on the overall SDG Index face

[1] For the list of indicators presented to the UN Statistical Commission visit: http://unstats.un.org/unsd/statcom/47th-session/documents/2016-2-IAEG-SDGs-E.pdf. As recognized by the IAEG-SDGs many of the proposed indicators currently lack data and some also lack statistical definitions.

major challenges on some goals. Both the SDG Index and Dashboard use the same metrics, though the methods of data analysis and aggregation vary, as described below.

We emphasize that the proposed SDG Index and SDG Dashboard are not official SDG indicators and complement the official monitoring processes launched by countries and the United Nations. Our focus is on identifying suitable metrics and data that allow countries to take stock of where they stand in 2016 with regards to achieving the SDGs and to identify priorities for early action. In parallel robust data production and management systems will need to be built in every country so that the 17 SDGs can be tracked with rigor.

Method

We apply three simple rules to determine suitable metrics for inclusion in the SDG Index and Dashboard. First, we identify technically sound quantitative metrics for each goal and associated sub-targets that are useful for policymaking, applicable to a broad range of country settings, constructed from well-established data sources, and – ideally – frequently updated. We have considered the indicators proposed to date by the IAEG-SDGs (other than the "policy indicators"[2]) as well as the indicators considered in SDSN (2015), which in turn draws on inputs from two broad public consultations. We have done our best to account for some of the new social targets, such as safety and wellbeing, by identifying rigorous, high-quality indicators from non-official sources, including the perception-based measure reported by Gallup and a measure of subjective wellbeing from Helliwell et al. (2015).

Second, we include only those technically-sound SDG indicators with sufficient up-to-date data at the country level. We acknowledge that using existing quantitative metrics means that some of the important new issues in the SDG agenda will not be measured adequately. As time will be required to collect the necessary data, this is inevitable using currently available datasets. We examined the potential indicators according to their coverage of the world's population focusing on the 154 UN member states with a national population greater than 1 million, a group of countries that includes more than 99 percent of the world population.[3] We included a candidate SDG indicator if recent comparable data exists for at least 80 percent of the 154 countries, i.e. at least 124 countries with a population greater than 1 million.[4] We are able to include 39 indicators in the current index (Table 1), between one and five variables per goal.[5] As the SDG data is improved over time, many more indicators will be added to the SDG Index and Dashboard.

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[2] Some indicators proposed by the IAEG-SDGs track the number of countries worldwide that have met certain policy benchmarks. Such indicators cannot be applied at the country level and are therefore not considered in the SDG index.

[3] Small countries, such as Small Island Developing States, face several unique development challenges. Among them are high fixed per capita costs for data collection, which generally results in lower data availability. Moreover, the small size of some countries' population makes it difficult to define representative survey samples required for household and other surveys. As a result, key MDG and SDG metrics remain unavailable in many small countries. This gap urgently needs to be filled with support from the international community.

[4] An exception is made for ocean-based indicators where we exclude landlocked countries from the minimum sample size resulting in 119 non-landlocked countries with a population greater than 1 million.

[5] SDG 12 presents major measurement challenges. We propose "anthropogenic wastewater that receives treatment (%)", which could equally be placed under SDG 6. Two frequently proposed indicators have adequate data coverage, but were excluded from the preliminary SDG index: (i) "Municipal Solid Waste (kg per capita per year)": this variable was excluded as it does not track the use and recycling of solid waste, i.e. whether waste management is sustainable; (ii) "Material Footprint (MF)" (currently being reviewed by the IAEG-SDGs): This indicator is available from UNEP (2016) for 186 countries and aggregates use expressed in tons across a heterogeneous set of materials (biomass, fossil fuels, metal ores, and non-metallic minerals), but without tracking recycling of these materials. As a result, the MF index does not track the re-use of materials and is therefore not well suited to measure the circular economy or sustainable consumption and production patterns. Moreover, its

Third, the SDG Index includes only those UN member states for which data is available for at least 80 percent of the indicators, i.e. at least 31 of 39 variables. Some small countries do have data on enough variables, so we were able to include 7 countries with a population of less than 1 million.[6] The SDG Dashboard (Table 7) currently presents OECD countries only, but will be expanded to cover all countries in future iterations of this work.

Since data included in the SDG Index is highly heterogeneous, we proceed in the following way to compute the aggregate SDG Index: countries are assigned a percentile rank on each variable. A percentile of 100 is assigned as the top score, and a percentile of $1/N_i$ denotes the lowest score, where N_i is the number of countries with data on the indicator *i*. The percentile rank for each variable is corrected for ties in the standard manner.[7]

To aggregate the percentile rank across all variables we first aggregate the percentile ranks for each SDG *j* and country *k* into I_{jk} and then across the 17 goals into I_k. Every SDG and the variables under each goal are given equal weight to reflect policymakers' commitment to treat all SDGs equally and as an "integrated and indivisible" set of goals (UN 2015, paragraph 5). This approach also has the advantage of allowing for the later addition of new variables for a particular SDG without affecting the relative weight of each SDG in the overall score.

As demonstrated by Rickels *et al.* (2014) for the case of the Ocean Health Index, the method for aggregating different variables into a single index can have profound implications on the overall ranking. To allow for maximum flexibility in aggregating data across each SDG *j,* we use the standard constant-elasticity-of-substitution (CES) function (Arrow et al. 1961, Blackorby and Donaldson 1982) to generate the SDG Index I_k.

$$I_k(N_k, I_{jk}, \rho) = \left[\sum_{j=1}^{N_k} \frac{1}{N_k} I_{jk}^{-\rho}\right]^{-\frac{1}{\rho}}$$

Where N_k denotes the number of SDGs for which country *k* has data available; I_{jk} is the percentile rank for SDG *j* and country *k*. The substitution parameter ρ describes the substitutability across components of the indicator with a permissible range of $-1 \leq \rho \leq \infty$ (Arrow et al. 1961). It yields the elasticity of substitution σ across components of the SDG Index:

$$\sigma = \frac{1}{1+\rho}$$

With $0 \leq \sigma \leq \infty$ and

$$\rho = \frac{1-\sigma}{\sigma}$$

aggregation of very different types of materials and their use into a single index makes the headline numbers hard to assess. For SDG 17, high-income countries are scored based on the amount of ODA they provide, whereas all other countries are scored based on government revenues as a percentage of GDP.

[6] These countries are: Bhutan, Cabo Verde, Iceland, Luxembourg, Malta, Montenegro and Suriname. We welcome suggestions for additional data sources to increase the number of countries included in the SDG Index.

[7] A percentile correction factor $PCF_i = 100*(N_i + 1 - R_i - RR_i)/(2*N_i)$ is subtracted from the percentile rank $P_i = 100*(RR_i+1)/N_i$, with R_i the rank of countries (top to bottom) and RR_i the reverse rank (bottom to top).

Three special cases of this CES function are frequently considered. First, if the components of the index are perfect substitutes (σ = ∞, ρ = -1) then a loss of one rank on one indicator (e.g. Gini coefficient) can be offset by a one-rank gain on another indicator (e.g. child mortality rate). The CES function with equal weights across components then assumes the form of the arithmetic mean:

$$I_k(N_k, I_{jk}) = \sum_{j=1}^{N_k} \frac{1}{N_k} I_{jk}$$

This case – sometimes referred to as "weak sustainability" – may be impossible for a range of reasons, including technical or ecological constraints or social preferences (Rickels *et al.* 2014). The assumption that all dimensions of the SDGs are perfect substitutes strikes us as implausible and counter to the integrated agenda that the SDGs seek to promote.

Strong sustainability occurs when the components of the SDG Index are not substitutable (σ = 0, ρ = ∞). In this case the CES function turns into a Leontief production function with orthogonal isoquants where the rank I_k of a country k is determined by the country's lowest rank I_{jk} across all SDGs j:

$$I_k(\ I_{jk}) = Min\{I_{jk}\}$$

Finally, an intermediate case of linear substitutability is given by the Cobb-Douglas production function with $\sigma = 1$ and $\rho = 1$. In this case the SDG Index I_k becomes the geometric mean of the goal indices I_{jk}:

$$I_k(N_j, I_{jk}) = \prod_{j=1}^{N_k} \sqrt[N_k]{I_{jk}}$$

The geometric mean is often used to aggregate heterogeneous variables with limited substitutability and in cases where the focus of the analysis is on percentage changes instead of absolute changes. A prominent example is the Human Development Index (HDI), which changed its method of aggregation across three dimensions from arithmetic to geometric mean in 2010 (UNDP 2015).

All three methods of combining indices for individual SDGs into one composite SDG Index are presented in Table 5. Results are compared across methods to determine the sensitivity of the composite SDG Index to different specifications.

In its present form, the SDG Index comprises between one and five variables per goal for each country. This represents too few variables to employ a nested CES function for creating the country rank for each SDG (I_{jk}). When there is more than one indicator for a goal, the country's percentile for that SDG can be estimated using the arithmetic average, the geometric mean, or the Leontief production function. After careful consideration of the three options, we prefer the arithmetic mean to calculate the percentile ranking for each SDG, because each SDG generally describes complementary policy priorities. The geometric mean and Leontief production function are only used in a second step, to calculate the composite SDG Index. Therefore, a country's percentile rank for each SDG is estimated as:

$$I_{jk}(N_{jk}, I_{ijk}) = \sum_{i=1}^{N_{jk}} \frac{1}{N_{jk}} I_{ijk}$$

Where N_{jk} denotes the number of indicators with data for SDG *j* and country *k*, and I_{ijk} is the rank of country *k* for variable *i* under SDG *j*.

The SDG Dashboard uses the raw data, color-coding each country based on the thresholds for each SDG indicator (Table 4). Variables are marked as green if the country has already achieved the SDG threshold or if little extra effort is required. Yellow coding denotes indicators for which significant extra effort is required. Finally, variables are marked in red if the country is far from achieving the SDG benchmarks. Where data is missing for all indicators related to a given goal, a blank (white) is returned. The ocean goal does not apply to landlocked countries, in which case "n/a" is shown.

While the SDG Index compares average performance across countries, the purpose of the SDG Dashboard is to identify the policy areas where major progress is needed at the national level. Scores are aggregated for each goal and country using the arithmetic mean. To this end we assign 0 to red scores, 1 to yellow ones, and 2 to green ones. Results are rounded to the nearest integer.

Alternatively the Leontief production function could be used to yield the score of the worst-performing variable for each country and goal. For example, if a goal has three indicators and the country scores green, yellow, and red score for each respectively, then the overall score for the goal would be red. Consequently, a red score on such an SDG Dashboard would show whether a country requires major effort on at least one variable for a given goal.

Data

We have consulted a large number of official and scientific data sources for the preliminary SDG Index and the Dashboard, including the World Development Indicator database (World Bank 2016), the Human Development Report (UNDP 2015), and OECD Stats (OECD 2016). Table 1 shows the variables that have been included in the Global SDG Index and Dashboard. All data used for the SDG Index and Dashboard is provided online in Excel format at *http://unsdsn.org/resources/publications/sdg-index/.*

Where necessary, we interpolate missing variables using data from earlier years, as described in the supplemental material. We have further imputed the following missing data based on trends observed in the data:

- "Poverty headcount ratio at $1.90 a day (2011 PPP) (% of population)": a value of 0% was assumed for all high-income countries where data was missing.
- "Prevalence of undernourishment (% of population)": a value of 0% was assumed for all high-income countries where data was missing.
- "Research and development expenditure (% of GDP)": a value of 0% was assumed for all low- and lower-middle income countries where data was missing.

As described in the final section, the preliminary SDG Index and Dashboard contain two aggregate indices. Such indices may exhibit collinearity with other variables, which is a particular concern with the Ocean Health Index as it contains a large number of component variables. These questions will be considered carefully during the public consultation of this draft paper.

Table 1. Indicators used in the preliminary Global SDG Index and SDG Dashboard

SDG	Description/Label	Year(s)*	Source
1	Poverty headcount ratio at $1.90 a day (2011 PPP) (% of population)	2009-2013	World Bank (2016)
2	Prevalence of undernourishment (% of population)	2013	World Bank (2016)
	Cereal yield (kg/ha)	2013	World Bank (2016)
3	Mortality rate, under-5 (per 1,000 live births)	2013	World Bank (2016)
	Healthy life expectancy at birth, total (years)	2013	WHO (2016a)
	Subjective Wellbeing (average ladder score)	2014	Helliwell *et al.* (2015)
	Physician density (per 1,000 people)	2004-2013	WHO (2016a)
	Adolescent fertility rate (births per 1,000 women ages 15-19)	2014	World Bank (2016)
4	Expected years of schooling	2014	UNDP (2015)
	Literacy rate of 15-24 year olds, both sexes, percentage	2001-2013	UNESCO (2015)
	Net primary enrolment rate (%)	1997-2014	WEF (2015)
5	Proportion of seats held by women in national parliaments (%)	2012-2104	World Bank (2016)
	Gender Inequality Index	2014	UNDP (2015)
6	Improved water source (% of population with access)	2011-2015	World Bank (2016)
	Improved sanitation facilities (% of population with access)	2011-2015	World Bank (2016)
	Water Stress Score	2013	Gasset *et al.* (2013)
7	Access to electricity (% of population)	2012	World Bank (2016)
	Alternative and nuclear energy (% of total energy use)	2012-2013	World Bank (2016)
8	GDP Growth Deviation	2001-2014	IMF (2015)
	Unemployment (% of total labor force)	2009/2014	Computed: based on IMF (2015)
9	Mobile broadband subscriptions per 100 inhabitants	2012-2015	ITU (2015)
	Percentage of population using the internet	2014	ITU (2015)
	Research and development expenditure (% of GDP)	2005-2012	UNDP (2015)
10	Gini index	2003-2012	World Bank (2016)
11	Mean annual concentration of PM2.5 in urban areas	2013	World Bank (2016)
12	Anthropogenic wastewater that receives treatment (%)	2012	Malik (2013)
13	CO2 emissions/GDP, PPP (tCO2/'000$)	2012	IEA (2014)
	CO2 emissions per capita (tCO2/capita)	2011	World Bank (2016)
14	Ocean Health Index	2015	Conservation International (2015)
	Percentage of marine sites important to biodiversity that are completely protected	2013	Butchart et al. (2015)
15	Weighted Red List Change per year	2014	Rodrigues *et al.* (2014)
	Annual change in forest area (%)	2001/2014	YCELP & CIESIN (2014)
	Percentage of terrestrial sites important to biodiversity that are completely protected	2013	Butchart et al. (2015)
16	Homicides per 100,000 population	2008-2012	UNDP (2015)
	Prison population per 100,000 people	2002-2013	UNDP (2015)
	Proportion of the population who feel safe walking alone at night in the city or area where they live.	2006-2015	Gallup (2015)
	Corruption Perceptions Index	2015	Transparency International (2014)
17	For high-income and all OECD DAC countries: International concessional public finance, including official development assistance (% GNI)	2013	OECD (2016)
	For all other countries: Government revenue (% of GDP)	2013	World Bank (2016)
	Health, Education and R&D spending (%GDP)	2005-2014	UNDP (2015)

* Data for the latest available year is used.

** As explained in footnote 5 this indicator addresses only parts of SDG 12 and could also fit under SDG 6. Suggestions for better tracking SDG 12 are particularly welcome.

Based on the methodology described above, we were able to include 147 of the 193 UN member states in this preliminary Global SDG Index. The 46 countries listed in Table 2 have been excluded for now from the SDG Index based on missing data. They comprise 32 small countries with populations of less than 1 million people and divide as follows by World Bank income group[8]: 11 high-income countries, 15 upper-middle-income countries, 11 lower-middle-income countries, and 9 low-income countries. Many of these countries face major challenges in achieving the SDGs, so investing in their capacity to generate high-quality data is a priority for early action on the goals.

Table 2. Countries not included in preliminary Global SDG Index due to insufficient data availability

Country	Missing Values	Country	Missing Values	Country	Missing Values
Afghanistan	27%	Grenada	51%	Sao Tome and Principe	31%
Andorra	54%	Guinea-Bissau	28%	Seychelles	31%
Antigua and Barbuda	46%	Guyana	23%	Solomon Islands	36%
Bahamas, The	38%	Kiribati	49%	Somalia	49%
Barbados	36%	Korea, Dem. Rep.	41%	South Sudan	51%
Belize	28%	Libya	31%	St. Kitts and Nevis	62%
Brunei Darussalam	38%	Liechtenstein	73%	St. Lucia	54%
Burundi	24%	Maldives	36%	St. Vincent and the Grenadines	51%
Central African Republic	24%	Marshall Islands	56%	Timor-Leste	28%
Comoros	28%	Micronesia, Fed. Sts.	49%	Tonga	51%
Cuba	23%	Monaco	56%	Turkmenistan	38%
Djibouti	28%	Nauru	n/a*	Tuvalu	72%
Dominica	54%	Palau	54%	Uzbekistan	23%
Equatorial Guinea	41%	Papua New Guinea	36%	Vanuatu	36%
Eritrea	28%	Samoa	44%		
Fiji	36%	San Marino	69%		

Source: authors' calculations

* Nauru was excluded from the analysis as the World Development Indicators database and most other datasets do not contain any data for the country.

The Global SDG Dashboard identifying priority challenges in every country will be presented in later iterations of this paper. This will require, *inter alia*, careful verification of appropriate thresholds for each indicator with the corresponding expert communities.

In view of the severe limitations to data availability across UN member states, we also created an SDG Index for OECD countries that draws on a set of 38 internationally comparable indicators. Additional indicators not included in the Global SDG Index are largely drawn from OECD statistics (OECD 2016). A second important motivation for creating an index and dashboard for OECD countries is to take a more granular look at these countries' challenges in meeting the SDGs. The variables included in this index are summarized in Table 3. Note that we replace a number of variables from the Global SDG Index, such as the incidence of extreme income poverty, with variables that allow for greater differentiation within the set of OECD countries. All 34 OECD member countries have data for at least 80% of the variables considered in this index. We use the same data for the OECD SDG Dashboard and apply thresholds identified in Table 4.

[8] Country income classifications correspond to the World Bank definitions as of 2015 (World Bank 2015)

Table 3. Indicators used in the preliminary OECD SDG Index and OECD SDG Dashboard.

SDG	Description/Label	Year(s)*	Source
1	Poverty rate after taxes and transfers, Poverty line 50%	2011-2014	OECD (2016)
2	Prevalence of obesity, BMI ≥ 30 (% of adult population)	2014	WHO (2016b)
	Cereal yield (kg/ha)	2013	World Bank (2016)
3	Healthy life expectancy at birth, total (years)	2013	WHO (2016a)
	Physician density (per 1,000 people)	2004-2013	WHO (2016a)
	Subjective Wellbeing (average ladder score)	2014	Helliwell et al. (2015)
4	Expected years of schooling	2014	UNDP (2015)
	Population aged 25-64 with tertiary education (%)	2011	OECD (2016)
	PISA score	2012	OECD (2016)
5	Proportion of seats held by women in national parliaments (%)	2012-2104	World Bank (2016)
	Gender wage gap (Total, % of male median wage)	2012	OECD (2016)
	Gender Inequality Index	2014	UNDP (2015)
6	Water Stress Score	2013	Gasset et al. (2013)
7	Alternative and nuclear energy (% of total energy use)	2012-2013	World Bank (2016)
8	Unemployment (% of total labor force)	2001-2014	IMF (2015)
	Real GDP Growth (%)	2012	OECD (2016)
9	Mobile broadband subscriptions per 100 inhabitants	2012-2015	ITU (2015)
	Percentage of population using the internet	2014	ITU (2015)
	Patent applications filed under the PCT in the inventor's country of residence	2012	Computed: based on OECD (2016)
	Research and development expenditure (% of GDP)	2005-2012	UNDP (2015)
10	PISA Social Justice Index	2015	Kroll (2015)
	Gini index	2003-2012	World Bank (2016)
11	Mean annual concentration of PM2.5 in urban areas	2013	World Bank (2016)
	Rooms per person	2001-2013	OECD (2016)
12	Municipal Waste Recycled (%)	2009-2013	OECD (2016)
	Anthropogenic wastewater that receives treatment (%)	2012	Malik (2013)
13	CO2 emissions/GDP, PPP (tCO2/'000$)	2012	Computed: based on IEA (2014)
	CO2 emissions per capita (tCO2/capita)	2011	World Bank (2016)
14	Ocean Health Index	2015	Conservation International (2015)
	Percentage of marine sites important to biodiversity that are completely protected	2013	Butchart et al. (2015)
15	Weighted Red List Change per year	2014	Rodrigues et al. (2014)
	Annual change in forest area (%)	2001/2014	Hansen et al. (2013)
	Percentage of terrestrial sites important to biodiversity that are completely protected	2013	Butchart et al. (2015)
16	Homicides per 100,000 population	2008-2012	UNDP (2015)
	Prison population per 100,000 people	2002-2013	UNDP (2015)
	Proportion of the population who feel safe walking alone at night in the city or area where they live.	2006-2015	Gallup (2015)
	Corruption Perceptions Index	2015	Transparency International (2014)
17	For high-income and all OECD DAC countries: International concessional public finance, including official development assistance (% GNI)	2013	OECD (2016)

* Data for the latest available year is used.

Note: The proposed thresholds are indicative and in need of further refinement. We will publish the Global SDG Dashboard following careful validation of the thresholds by expert communities.

Table 4. Indicator thresholds used in OECD SDG Dashboard

SDG	Description/Label	Thresholds		
		Green	Yellow	Red
1	Poverty rate after taxes and transfers, Poverty line 50%	<10%	10% <= value <= 15%	>15%
2	Prevalence of obesity, BMI ≥ 30 (% of adult population)	<10%	10% <= value <=25%	>25%
	Cereal yield (kg/ha)	>2.5	1.5 <= value <= 2.5	<1.5
3	Healthy life expectancy at birth, total (years)	>65	60 <= value <= 65	<60
	Physician density (per 1000 people)	>3	1 <= value <= 3	<1
	Subjective Wellbeing (average ladder score)	>6	5 <= value <= 6	<5
4	Expected years of schooling	>15	12 <= value <= 15	<12
	Population aged 25-64 with tertiary education (%)	>25%	15% <= value <= 25%	<15%
	PISA score	>493	400 <= value <= 493	<400
5	Proportion of seats held by women in national parliaments (%)	>40%	20% <= value <= 40%	<20%
	Gender wage gap (Total, % of male median wage)	<7.5%	7.5% <= value <= 15%	>15%
	Gender Inequality Index	<.2	.2 <= value <= .4	>.4
6	Water Stress Score	<2	2 <= value <= 3.5	>3.5
7	Alternative and nuclear energy (% of total energy use)	>30%	15% <= value <= 30%	<15%
8	Unemployment (% of total labor force)	<5%	5% <= value <= 10%	>10%
	GDP Growth	>2	1 <= value <= 2	<1
9	Mobile broadband subscriptions per 100 inhabitants	>75%	50% <= value <= 75%	<50%
	Percentage of population using the internet	>80%	50% <= value <= 80%	<50%
	Patent applications filed under the PCT in the inventor's country of residence (per million people)	>100	50 <= value <= 100	<50
	Research and development expenditure (% of GDP)	>1.5%	0.5% <= value <= 1.5%	<0.5%
10	PISA Social Justice Index	>5.6	4 <= value <= 5.6	<4
	Gini index	<35	35 <= value <= 45	>45
11	Rooms per person	>1.5	1 <= value <= 1.5	<1
	Mean annual concentration of PM2.5 in urban areas	<10	10 <= value <= 20	>20
12	Percentage of waste recycled	>40%	20% <= value <= 40%	<20%
	Percentage of anthropogenic wastewater that receives treatment (%)	>50%	15% <= value <= 50%	<15%
13	CO2 emissions/GDP, PPP (tCO2/'000$)	<.1	.1 <= value <= 0.3	>.3
	CO2 emissions per capita (tCO2/capita)	<3	3 <= value <= 8	>8
14	Ocean Health Index	>60	50 <= value <= 60	<50
	% of marine sites important to biodiversity that are completely protected	>50%	10% <= value <= 50%	<10%
15	Weighted Red List Change per year	>0	-1 <= value <= 0	< -1
	Annual change in forest area (%)	>0	-2 <= value <= 0	<-2
	% of terrestrial sites important to biodiversity that are completely protected	>50%	10% <= value <= 50%	<10%
16	Homicides per 100,000 population	<1.5	1.5 <= value <= 3	>3
	Prison population per 100,000 people	<100	100 <= value <= 200	>200
	Proportion of the population who feel safe walking alone at night in the city or area where they live.	>80%	50% <= value <= 80%	<50%
	Corruption Perception Index	>70	50 <= value <= 70	<50
17	For high-income and all OECD DAC countries: International concessional public finance, including official development assistance (% GNI)	>0.7%	0.35% <= value <= 0.7%	<0.35%

Source: expert opinion, normative thresholds and authors' assessment.
Note: The proposed thresholds are indicative and in need of further refinement. We will publish the Global SDG Dashboard following careful validation of the thresholds by expert communities.

Results

Table 5 presents the preliminary SDG Index and country rankings for the three aggregation methods considered: (A) arithmetic mean (weak sustainability with infinite substitution), (G) geometric mean (linear elasticity of substitution), and (L) Leontief production function (hard sustainability with no substitution).

Table 5. Preliminary Global SDG Index: Global ranking and score by country and aggregation method

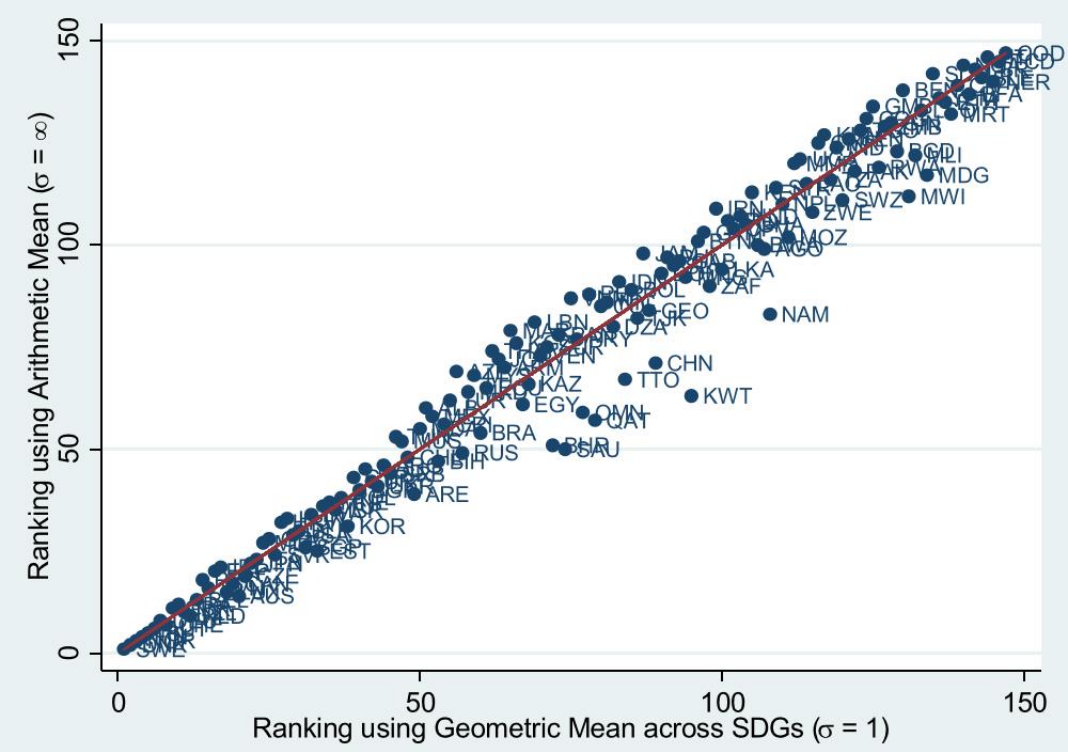
Country	Code	(G)	(A)	(L)	Country	Code	(G)	(A)	(L)
Sweden	SWE	1	1	1	Serbia	SRB	45	44	59
Denmark	DNK	2	2	7	Tunisia	TUN	46	53	9
Norway	NOR	3	3	6	Mauritius	MUS	47	52	31
Finland	FIN	4	4	28	Chile	CHL	48	48	70
Iceland	ISL	5	5	13	United Arab Emirates	ARE	49	39	104
Austria	AUT	6	6	11	Moldova	MDA	50	55	19
Germany	DEU	7	8	20	Albania	ALB	51	60	36
Switzerland	CHE	8	7	15	Mexico	MEX	52	58	46
United Kingdom	GBR	9	11	8	Bosnia and Herzegovina	BIH	53	47	71
France	FRA	10	12	4	Costa Rica	CRI	54	56	61
New Zealand	NZL	11	10	18	Turkey	TUR	55	62	22
Netherlands	NLD	12	9	33	Azerbaijan	AZE	56	69	30
Belgium	BEL	13	13	23	Russian Federation	RUS	57	49	108
Portugal	PRT	14	18	3	Macedonia, FYR	MKD	58	64	34
Slovenia	SVN	15	16	25	Malaysia	MYS	59	68	45
Spain	ESP	16	20	2	Brazil	BRA	60	54	88
Ireland	IRL	17	21	10	Ecuador	ECU	61	65	58
Luxembourg	LUX	18	15	48	Thailand	THA	62	74	38
Canada	CAN	19	17	65	Jordan	JOR	63	72	50
Australia	AUS	20	14	72	Armenia	ARM	64	70	62
Czech Republic	CZE	21	19	63	Morocco	MAR	65	79	27
Japan	JPN	22	22	37	Kyrgyz Republic	KGZ	66	76	42
Italy	ITA	23	23	21	Egypt, Arab Rep.	EGY	67	61	92
Malta	MLT	24	27	12	Kazakhstan	KAZ	68	66	109
Greece	GRC	25	28	32	Lebanon	LBN	69	81	43
Slovak Republic	SVK	26	24	17	Venezuela, RB	VEN	70	73	78
Croatia	HRV	27	32	24	Suriname	SUR	71	75	84
Hungary	HUN	28	33	5	Bahrain	BHR	72	51	117
United States	USA	29	29	60	Panama	PAN	73	78	69
Belarus	BLR	30	30	49	Saudi Arabia	SAU	74	50	138
Singapore	SGP	31	26	35	Vietnam	VNM	75	87	47
Latvia	LVA	32	34	39	Paraguay	PRY	76	77	74
Estonia	EST	33	25	95	Oman	OMN	77	59	89
Lithuania	LTU	34	36	41	Peru	PER	78	88	54
Montenegro	MNE	35	37	16	Qatar	QAT	79	57	102
Israel	ISR	36	35	52	Colombia	COL	80	85	101
Poland	POL	37	38	55	Nicaragua	NIC	81	86	66
Korea, Rep.	KOR	38	31	67	Algeria	DZA	82	80	110
Cyprus	CYP	39	43	26	Indonesia	IDN	83	91	73
Bulgaria	BGR	40	40	53	Trinidad and Tobago	TTO	84	67	137
Romania	ROU	41	45	14	Bolivia	BOL	85	89	105
Uruguay	URY	42	42	44	Tajikistan	TJK	86	82	96
Ukraine	UKR	43	41	64	Jamaica	JAM	87	98	40
Argentina	ARG	44	46	29	Georgia	GEO	88	84	85

Country	Code	(G)	(A)	(L)	Country	Code	(G)	(A)	(L)
China	CHN	89	71	142	India	IND	119	124	127
Dominican Republic	DOM	90	93	79	Swaziland	SWZ	120	111	86
Philippines	PHL	91	97	56	Senegal	SEN	121	126	106
El Salvador	SLV	92	95	57	Pakistan	PAK	122	118	123
Gabon	GAB	93	96	80	Togo	TGO	123	128	107
Mongolia	MNG	94	92	68	Congo, Rep.	COG	124	131	97
Kuwait	KWT	95	63	133	Gambia, The	GMB	125	134	91
Bhutan	BTN	96	101	80	Rwanda	RWA	126	119	119
Guatemala	GTM	97	103	94	Zambia	ZMB	127	129	124
South Africa	ZAF	98	90	129	Guinea	GIN	128	130	112
Iran, Islamic Rep.	IRN	99	109	76	Bangladesh	BGD	129	123	131
Sri Lanka	LKA	100	94	121	Benin	BEN	130	138	93
Iraq	IRQ	101	106	83	Malawi	MWI	131	112	134
Cabo Verde	CPV	102	104	114	Mali	MLI	132	122	120
Honduras	HND	103	107	113	Lesotho	LSO	133	133	115
Ghana	GHA	104	105	90	Madagascar	MDG	134	117	145
Kenya	KEN	105	113	51	Sudan	SDN	135	142	126
Botswana	BWA	106	100	111	Yemen, Rep.	YEM	136	136	140
Angola	AGO	107	99	98	Ethiopia	ETH	137	135	146
Namibia	NAM	108	83	135	Mauritania	MRT	138	132	147
Syrian Arab Republic	SYR	109	114	77	Cote d'Ivoire	CIV	139	139	136
Nepal	NPL	110	110	118	Nigeria	NGA	140	144	128
Mozambique	MOZ	111	102	116	Burkina Faso	BFA	141	137	125
Myanmar	MMR	112	120	80	Liberia	LBR	142	143	132
Uganda	UGA	113	121	99	Sierra Leone	SLE	143	141	141
Lao PDR	LAO	114	115	103	Haiti	HTI	144	146	122
Zimbabwe	ZWE	115	108	130	Niger	NER	145	140	144
Cameroon	CMR	116	125	87	Chad	TCD	146	145	143
Cambodia	KHM	117	127	100	Congo, Dem. Rep.	COD	147	147	139
Tanzania	TZA	118	116	75					

Source: authors' calculations

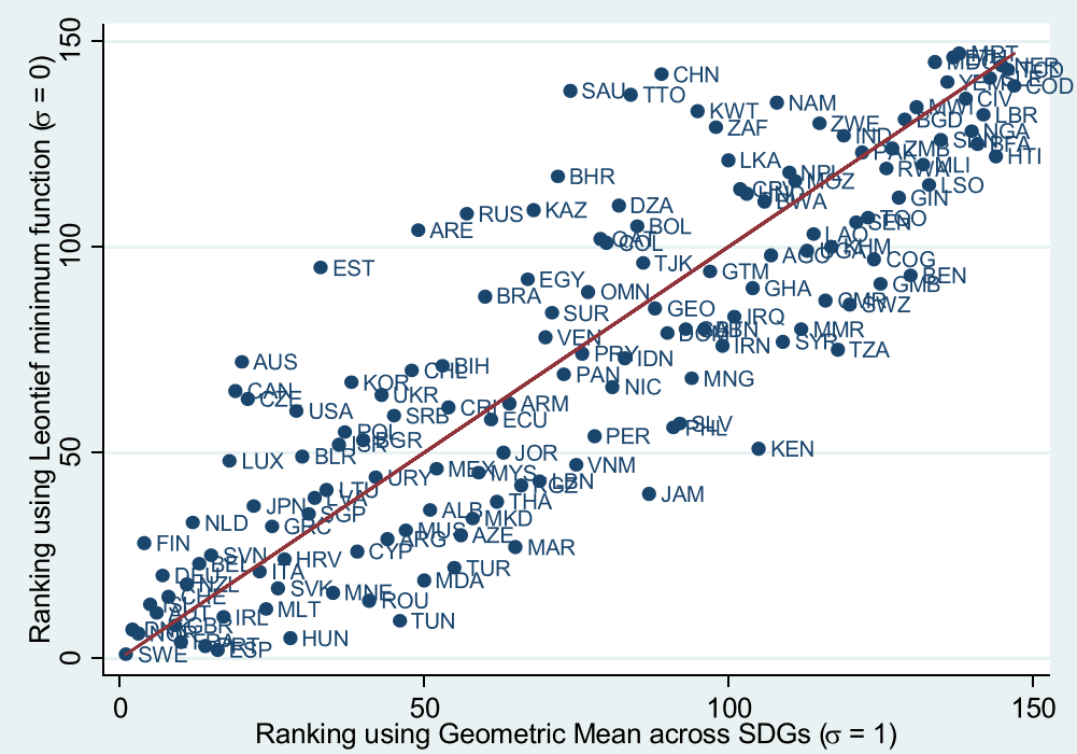
Figure 1 compares the SDG Indices obtained by aggregation using geometric ($\sigma = 1$) and arithmetic ($\sigma = \infty$) means. Both indices show a high degree of correlation with a Spearman rank order correlation (Zwillinger and Kokoska 2000) of 0.98. In contrast, an SDG Index constructed using the Leontief production function ($\sigma = 0$) exhibits considerably higher variation (Figure 2) and generates a Spearman rank order correlation with the linearly elastic SDG Index of 0.85.

Figure 1. Comparison of Global SDG Indices using geometric mean and arithmetic mean



Source: authors' calculations

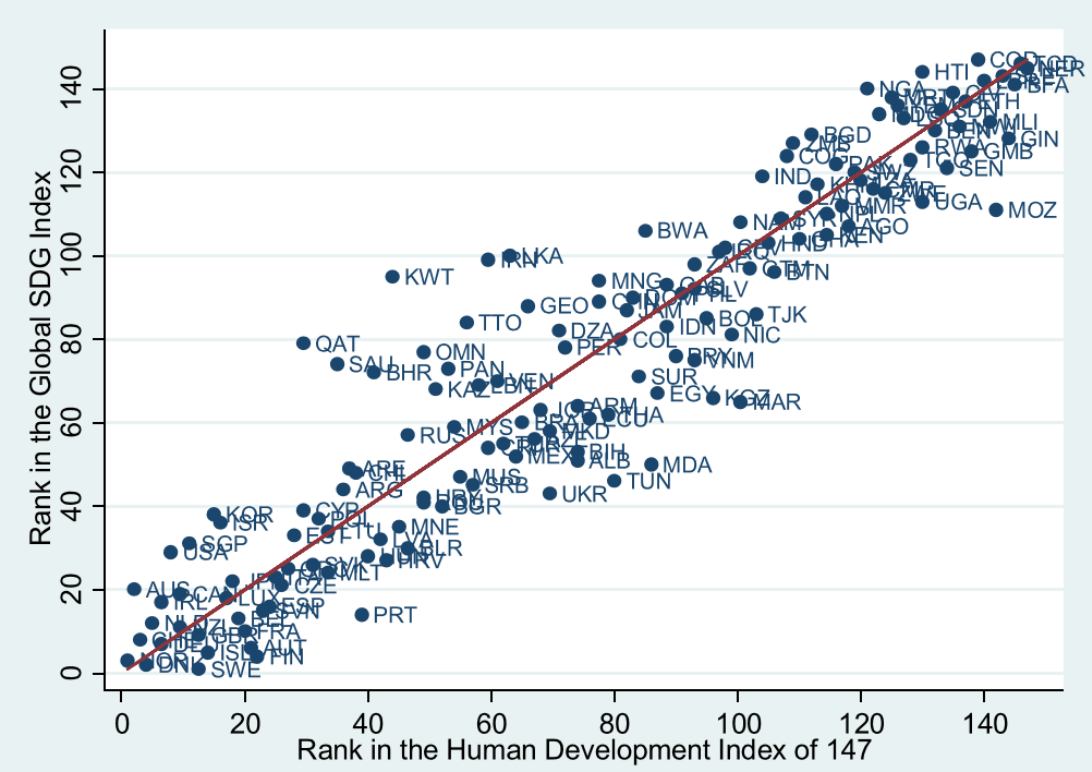
Figure 2. Comparison of Global SDG Indices using geometric mean and Leontief Production function



Source: authors' calculations

Figure 3 compares the Global SDG Index with the country ranking by the Human Development Index (UNDP 2015). For this chart we have re-ranked the HDI for the 147 countries included in the preliminary Global SDG Index. The comparison reveals a high degree of correlation with significant differences for some countries.

Figure 3. Comparison of rankings by Global SDG Index and by Human Development Index



Source: UNDP (2015) and authors' calculations

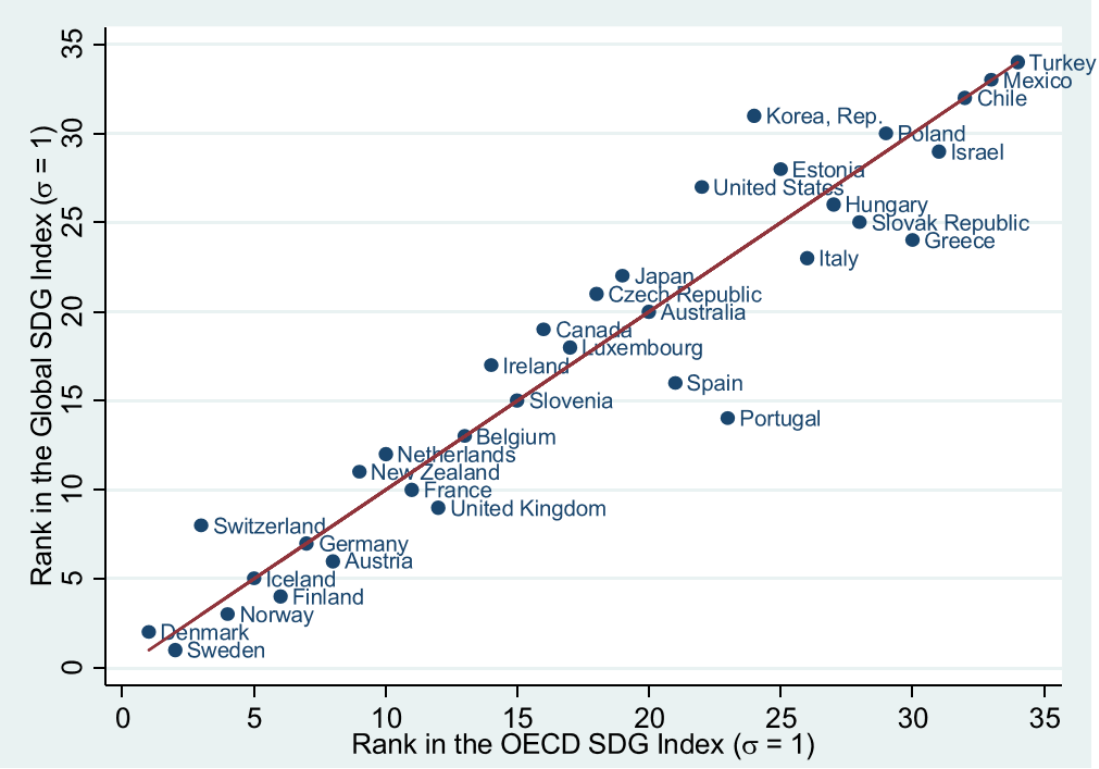
The OECD SDG Index covers 34 countries and is presented in Table 6. Figure 4 compares the ranking of OECD countries in both indexes using linearly elastic substitution (σ = 1). A Spearman rank order correlation for OECD countries in the global and the OECD SDG Index returns a value of 0.95, confirming that the two indices are very well aligned.

Table 6. Preliminary OECD SDG Index: OECD ranking and score by country and aggregation method

Country	(G)	(A)	(L)	Country	(G)	(A)	(L)
Denmark	1	2	8	Czech Republic	18	20	17
Sweden	2	1	4	Japan	19	19	15
Switzerland	3	5	3	Australia	20	18	24
Norway	4	3	11	Spain	21	22	27
Iceland	5	4	16	United States	22	24	21
Finland	6	6	6	Portugal	23	26	24
Germany	7	8	1	Korea, Rep.	24	21	22
Austria	8	9	5	Estonia	25	23	22
New Zealand	9	10	2	Italy	26	28	18
Netherlands	10	7	20	Hungary	27	27	14
France	11	11	9	Slovak Republic	28	25	26
United Kingdom	12	12	7	Poland	29	30	27
Belgium	13	13	10	Greece	30	31	32
Ireland	14	15	12	Israel	31	29	32
Slovenia	15	16	13	Chile	32	32	31
Canada	16	17	18	Mexico	33	33	30
Luxembourg	17	14	32	Turkey	34	34	29

Source: authors' calculations

Figure 4. Rank comparison for OECD countries in global and OECD SDG Indices



Source: authors' calculations

We propose an indicative OECD SDG Dashboard (Table 7) using the data available for OECD countries. This Dashboard shows that even the highest performing OECD countries will face major challenges in achieving some of the SDGs. We underscore the preliminary nature of these findings and welcome suggestions for how to improve the SDG Dashboard. The Global SDG Dashboard will be presented at a later stage once the thresholds have been carefully reviewed.

Table 7. Dashboard for OECD countries using indicators of the OECD SDG Index

Country	SDG1	SDG2	SDG3	SDG4	SDG5	SDG6	SDG7	SDG8	SDG9	SDG10	SDG11	SDG12	SDG13	SDG14*	SDG15	SDG16	SDG17
Australia																	
Austria														n/a			
Belgium																	
Canada																	
Chile																	
Czech Republic														n/a			
Denmark																	
Estonia																	
Finland																	
France																	
Germany																	
Greece																	
Hungary														n/a			
Iceland																	
Ireland																	
Israel																	
Italy																	
Japan																	
Korea, Rep.																	
Luxembourg														n/a			
Mexico																	
Netherlands																	
New Zealand																	
Norway																	
Poland																	
Portugal																	
Slovak Republic														n/a			
Slovenia																	
Spain																	
Sweden																	
Switzerland														n/a			
Turkey																	
United Kingdom																	
United States																	

Source: authors' calculations

Note: The complete SDG OECD Dashboard, disaggregated by indicator, is available in the supplemental materials.

* SDG14 is not applicable to landlocked countries, marked (n/a).

Discussion and next steps

To our knowledge, the preliminary SDG Index and SDG Dashboard represent the first attempt to measure the baseline of SDG achievement across all 17 goals for both developed and developing countries. Unfortunately, the limited data availability makes it impossible at this time to construct an SDG Index and Dashboard that track the SDGs comprehensively.

During the public consultation we would be particularly grateful for help and advice on how to fill some of the following major data gaps in the preliminary SDG Index and Dashboard:

- Sustainable agriculture (SDG 2)
- Universal health coverage (SDG 3)
- Quality of education (SDG 4)
- Broader measures of women empowerment (SDG 5)
- Integrated water resources management (SDG 6)
- Access to modern cooking fuels (SDG 7)
- Decent work (SDG 8)
- Inclusive and sustainable cities (SDG 11)
- Sustainable consumption and production (SDG 12)
- Climate change impacts (SDG 13)
- Ecosystem services (SDGs 14 and 15)
- Means of implementation (SDG 17).

Moreover, available time series data is too sparse to estimate country-level rates of change for a sufficient number of variables. As a result, the SDG Index and Dashboard provide an initial snapshot of where countries stand with regards to the SDGs. Future work should focus on filling gaps in available data and estimating historic baselines to compute rates of change.

In this preliminary form, the SDG Index and Dashboard comprise the Ocean Health Index for goal 14 and the Gender Inequality Index for Goal 5, which are in turn aggregate indices. Both may raise issues of collinearity with the aggregate SDG Index, and the Ocean Health Index comprises many modeled variables. Collinearity and inclusion of modeled parameters will require careful consideration in the next iteration of the SDG Index and Dashboard.

“Weak sustainability”, i.e. perfect substitutability across variables and goals, represents an implausibly strong assumption with regards to the SDGs since the SDGs were designed to ensure parallel progress on all three dimensions of sustainable development (UN 2015). Countries need to accelerate progress towards economic, social, and environmental goals in parallel since these all reinforce one another. For this reason the arithmetic mean ($\sigma = \infty$) is a poor methodology for aggregating indicators across the SDGs. As a tool for policy analysis, it would also skew attention to those SDGs where progress can be accelerated most easily, at the expense of those that require greater transformations – irrespective of their relative contribution towards holistic sustainable development.

On the other hand, the Leontief production function yields a country index that relies solely on the SDG where the country ranks poorest. This generates high sensitivity to measurement errors in a single data point. Similarly, a country may perform particularly poorly on one SDG due to exogenous constraints that are beyond its control and might be alleviated through international trade. For example, it would seem unreasonable to base an arid

country's SDG score entirely on water stress, particularly if some of this stress can be alleviated through international trade.

On balance we therefore recommend the linearly elastic geometric mean ($\sigma = 1$) for aggregating the SDGs index. This method takes account of the limited substitutability between SDGs and ensures the equal consideration of all goals.

Comments on earlier versions of this draft paper have focused on whether the proposed Global SDG Index adequately captures the transformative nature of the SDGs as intended by UN member states (UN 2015). In particular, concerns have been raised about the significant correlation between the Global SDG Index and the HDI or other common metrics for human development that exclude environmental considerations and tend to rank rich countries highest (Figure 3). This correlation is robust to different specifications of the Global SDG Index.

In parts, this correlation may result from the fact that the Global SDG Index lacks adequate measures for a number of SDG priorities, particularly relating to SDG 12 on sustainable consumption and production where richer countries tend to fare worse. On the other hand, higher-income countries tend to perform better on most economic and social SDG priorities. They also perform better on some environmental priorities, including access to wastewater treatment, deforestation rates, and rates of biodiversity loss. Rich countries perform worse on greenhouse gas emissions and some metrics for sustainable consumption and production, but these represent a modest share of SDG priorities. It therefore appears that the relatively higher ranking of richer countries is not primarily the result of poor or missing data.

The preliminary OECD SDG Dashboard demonstrates that even the wealthiest countries face major challenges on some SDGs, including goals that are not related to the environment. We underscore that the proposed thresholds are indicative and in need of further refinement. We will publish the Global SDG Dashboard following careful validation of the thresholds and aggregation methodology by expert communities.

In spite of the gaps and limitations described above, we believe the SDG Index and SDG Dashboard can be used as a first step for countries to assess their starting point on the SDGs. We hope they will foster discussion and further research on SDG priorities at national, regional, and global levels. The OECD SDG Index and Dashboard provide greater granularity for comparing high-income countries and includes several variables that should be considered for inclusion in the Global SDG Index and Dashboard as soon as better and more comprehensive data becomes available.

Thematic indices that track individual SDGs or key priorities will be an important complement to a Global SDG Index. The disaggregated SDG Indices provided in the supplemental online material constitute a first step towards such thematic indices. Each epistemic community may consider the feasibility and desirability of improved thematic SDG indices for its priorities.

We underscore the very preliminary nature of this work and invite suggestions and comments on this draft paper. In particular we urge researchers to share suggestions for additional data that meet the basic test of SDG relevance and cross-country up-to-date coverage. Comments and suggestions should be sent via the following online form. The SDSN will incorporate comments received by 31 March and will issue a revised SDG Index before the July 2016 meeting of the High-Level Political Forum, which reviews progress towards implementing the SDGs. We will also publish further country-level data on achievement of individual SDGs by that time, which can guide countries in identifying priorities for early action on the SDGs.

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